IC: Model

Stockholm PhD Minicourse 2023

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May 2023

A Little Formality

We need three “spaces” (“sets”):

1. An Underlying True Value, V
   - think this is what you will get if you pick correctly
   - revealed only at end of game (after action)
     - (PS: payoff itself could be noisy = signal)
2. A Private Signal (Opinion), S
   - we can allow occasional public signals, too
3. A Publicly Observable Action, A
   - “pick action inferred closest to true value”

True Value ("State")

- Assume the true value is bounded
  - what would an unbounded true value / payoff even mean?
- If agents are homogeneous, can make payoff the state
- Private signal will be drawn based on true value

(Private) Signal

- Just stated: must depend on true value
- Must be (reasonably) finite
  - must not have (non-zero-prob) signal perfectly informative about the underlying true value state (conditional?)
  - we could argue about what an epsilon probability of a perfect signal means, as agent/time goes to infinity
    - masochistic algebra fun, but really a distraction.

- signal could also be occasional, not given, asymmetric, endogenous, costly, etc.
    - signal type could be known or guessed by later agents.
    - many, many variations possible

Action

- The important endogenous choice, to be optimized by independent self-interested rational agents.
  - Everyone is selfish.
  - Everyone has same action choices (observable)
  - Bayesian Nash, but irrelevant.
    - Agents' payoffs do not depend on others' behavior.
    - There are no strategic player considerations.

- We need some tie-break rule if indifferent:
  - Follow Own (easy)
  - Follow Predecessor (easy)
  - Follow Waffle (shrink towards middle)
  - Follow Random (earliest papers, masochistic)

Queue Position

- Easiest: exogenous ordering in queue.
  - uses only one subscript!! ;-)
  - subscripts are expensive!
  - (endogenous delay will be interesting)

Definition

An agent is in an IC if her optimal action choice is independent of her information.

- all agents can be in a cascade forever,
- or just some for a while
  - (e.g., if the underlying value is drifting).

Primary Result

- Under *above* *assumptions*:

An IC will occur with probability 100%

- and it *very often* occurs *very rapidly*, too.
    - in card draw, prob(HH*) or prob(LL*).

Example: Welch 1992:

1. True value V is distributed uniform from 0 to 1.
2. Signal is symmetric H or L:

$$p(H|V) = 1 - p(L|V) = V.$$

   - if $V = 0.25$, p(H)=1/4, p(L)=3/4.
3. Action is *adopt* (A) or reject (R).
   - payoff(A | V>0.5) > payoff(R | V>0.5)
   - payoff(A | V<0.5) < payoff(R | V<0.5)

4. Original tie-break rule: flip 50-50 coin
    - here make it easier: just follow own signal.
    - in paper, algebra a little more "impressive" (for referee).

Conjugate Prior

- Canonical Bayesian example. Easy to work with!
- Bayes' Rule (love the guy!):

$$E(V|h\,H's, l\,L's) = \frac{h+1}{(h+l)+2}$$

  - $H : E(V|H) = 2/3$
  - $L : E(V|L) = 1/3$
  - $HH : E(V|HH) = 3/4$
  - $HL : E(V|HL) = E(V|LH) = 1/2$
  - $LL : E(V|LL) = 1/4$

- PS: Can integrate over prior uniform distribution

$$E(h\ H's \mid n \text{ draws}) = \frac{1}{n+1}$$

  - *as* likely to get (30 H's; 0 L's) *as* (10 H's; 20 L's)
  - used in paper for monopoly pricing *and* signaling, too.

What Choices?

- use '[AR]' for action, '[LH]' for signal

- $H : E(V|H) = 2/3 \Rightarrow A.$
- $AL : E(V|AL) = 1/2 \Rightarrow$ (Q: AR or AA?)
- $AH : E(V|AH) = \qquad \Rightarrow$
- $AAL : E(V|AAL) = \qquad \Rightarrow$
- $AA??????L : E(V|A^t, L) = \qquad \Rightarrow$

works same way in reverse with RH, etc.

IC Result: Prob of (Right or Wrong) IC

- Keep information state as sum of previous As minus sum of Rs. When $|\#A - \#R| \geq 2$, an IC ensues.
- Probability of getting two consecutive HHs or LLs
  - next one will be in IC

$$2 : p^2 + (1-p)^2 = 1 - 2p(1-p)$$

$$4 : p^2 + (1-p)^2 + 2p(1-p) \cdot (p^2 + (1-p)^2)$$

$$6 : [1 + 2p(1-p) + (2p(1-p))^2][p^2 + (1-p)^2]$$

$$(t-2)/2 \; : \; \left[\sum_{i=0}^{t} (2p(1-p))^t\right] [p^2 + (1-p)^2]$$

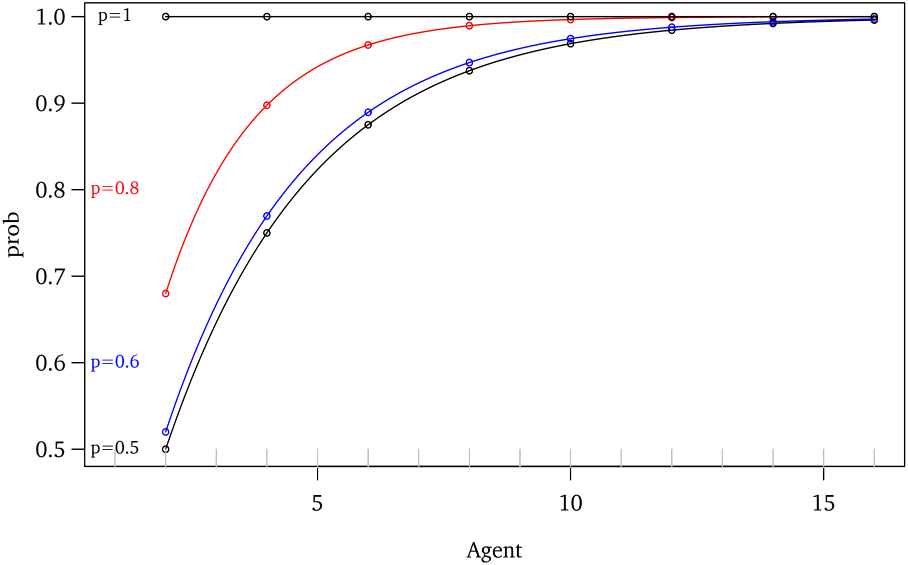
$$(T-2)/2 \; : \; \left[\frac{1-(2p(1-p))^{T+1}}{1-(2p(1-p))}\right] [p^2 + (1-p)^2]$$

```r
ff <- function( T )
  ( 1- (2*p*(1-p))^((T-2)/2+1) ) / ( 1-2*p*(1-p) ) *
  ( p^2 + (1-p)^2 )
```

(formula/figure works for even t only)

Quick Program Check

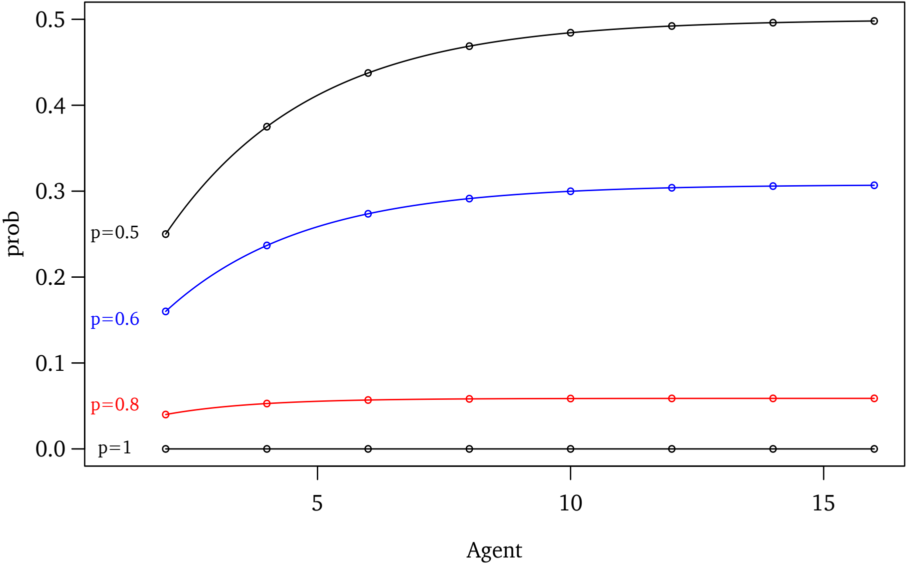
```
N <- 1000000; T <- 6; p <- 0.51

M <- matrix( rbinom(N*T, 1, p) , ncol=T )
ss <- seq(2, T, 2)
isoppeven <- function(v) all( v[ss] != v[ss-1] )

isnocascade <- apply( M, 1, FUN=isoppeven )

cat("see some sample draws:\n")
print(head( cbind(M, isnocascade) ))

cat("probability not in a cascade:\n")
print(mean(isnocascade))
```



p=1
p=0.8
p=0.6
p=0.5
1.0
0.9
0.8
0.7
0.6
0.5
prob
5
10
15
Agent

IC Result: Prob of Incorrect IC

- “Remove the p^2” in the T=2
- Will *asymptote* to finite number $< 1/2$



0.5
0.4
0.3
0.2
0.1
0.0
prob
p=0.5
p=0.6
p=0.8
p=1
5
10
15
Agent

Ex: Bikhchandani, Hirshleifer, Welch 1992

- Less fin-econ / more general econ than W 1992!
  - reason why ICs are so well known today
  - and what great coauthors are for!

BHW: Changes from W

1. True value is not uniform but discrete (G or B in ex).
2. Signal is monotonically informative (H or L in ex).
3. Still same tie-break rule: randomize.

4. Added fashion leaders (more info), fragility (to public information), and depth.
5. Added then removed pseudocascades.

Similar Probability Algebra

1. Up Cascade (Dn is the same)

$$\frac{1-(p-p^2)^{T/2}}{2}$$

2. No cascade

$$p-p^2$$

3. Correct cascade

$$\frac{p(p+1)\left[1-(p-p^2)^{T/2}\right]}{2(1-p+p^2)}$$

4. Incorrect cascade

$$\frac{(p-2)(p-1)\left[1-(p-p^2)^{T/2}\right]}{2(1-p+p^2)}$$

BHTW Bullet Points

- Conformity
- Idiosyncrasy / Path Dependence
- Information externality
- Fragility

Example: Banerjee 1992

- Much harder to explain.
  - Clearly independent work.
  - I did not know his, he did not know mine.
    - time before Internet (and undated)
    - zero inspiration or ancestry
    - citation to B came after R&R when we became aware

1. State space continuous, e.g., [0...1]
2. Action space continuous, e.g., [0...1]
3. Payoff
   - positive if *action* is perfectly correct
   - zero if *action* is epsilon off
4. Signal
   - signal either perfectly informative or uninformative
   - *agent* may know whether signal is uninformative,
   - **but** if he knows it is *a* signal, *agent* still does not know whether signal is useful or useless

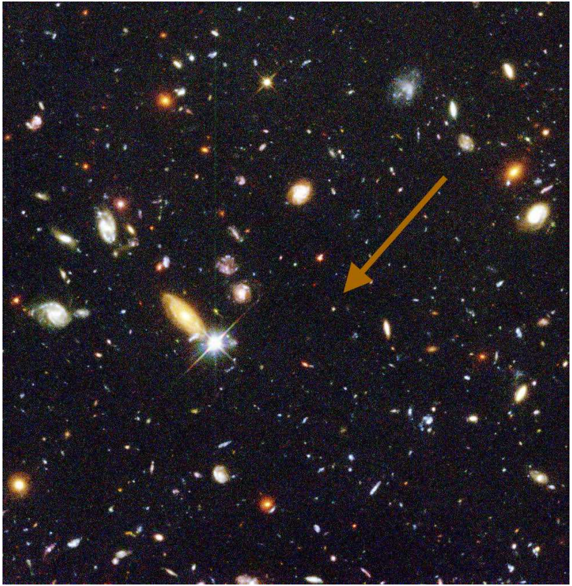
Result

- decision tree puzzle exercises
- at some point, agents without a signal then copy predecessor(s)
- because even agents with signal are not sure whether their signal was a real signal or a fake signal, so eventually they (usually) follow predecessors, too.

Rough Intuition

- Painful to sort out.
  - If I see choices $\{2/3, 1/\pi\}$ before me, and I have signal e^{-1}, I choose e^{-1} instead of $1/\pi$.
  - If I see choices $\{2/3, 2/3, 2/3\}$, and I have signal e^{-1}, I may choose 2/3.
  - If I see choices $\{2/3, 1/\pi, 1/\pi\}$, and I have precisely 2/3, I know I have a signal, I switch to correct IC.
    - #2: $1/\pi$ was probably random draw.
    - #3: was probably uninformed, just copied #2
    - me: only way to get 2/3 was **exact same** info
  - also cases where many agents get correct signal, but they all appeared after a^t, so they all ended up following wrong signal

There exists a world...



Banerjee

- Did have endogenous choice of non-use of private information = IC.
- Not general or (easily) generalizable
- Sort of *abandoned*, except in citations

- IMHO (Abhijit may disagree)
  - rarely read, often cited;
  - ... as having been like BHW.