IC: Actions

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Question

Do ICs occur with infinitely many relevant actions?

Action Discreteness

- As $C \to \infty$, ICs disappear
- Intuitively action discreteness is at the heart of ICs
  - Obvious and easy with two action choices
- Infinitely fine action choices destroy responsiveness (Ali), "invertibility"
  - In the extreme: continuous relevant choices

Key Question

- Quantitative, not qualitative
- Can ICs then still matter economically? Or are they "just" a two-action curiosity?

Economic Meaning of Action Discreteness

- No one buys 3.1415 apples
- Even if you buy 245g of Herring and your predecessor bought 240g of Herring, this difference probably would not even register.
- In real life, basic joining or not joining is often easiest to observe and remember.
  - If ICs just matter for two actions, we better be *very* cautious about interpreting economic meaning
- How much do ICs matter **quantatively**?

How To Model?

- Want model with more vs. less choice granularity
  - ideally also in the very long-run.
  - ergo, not ideal with just two underlying **value states**
  - in this case, only two extremes are asymptotically optimal.
    - by asymptotic, I mean infinitely many signals.
- Want **relevant** action choices for granularity
  - not choices of 0.001, 0.002, 0.50, 0.998, 0.999 ?

Used Baseline: Welch 1992 model

- Uniform distribution of possible values
  - "Diffuse Bayesian prior"
  - Asymptotically, not just V=0 vs. V=1, but continuous V
- Binary signals, H, L, with probability p=V.
  - makes tracking decisions a lot easier!
- Easy non-abstract inference rule with signals:

$$EV(h,S) = (h+1)/(S+2)$$

- Nothing IC for a while. Observe previous signals.

Relevant Available Choices

- 2: 1/3, 2/3
- 3: 1/4, 2/4, 3/4
- S: $(i+1)/(S+1)$ for $i \in [0,\ldots,S-1]$

- matches possible inferences nicely.
- more *relevant* actions = least cascade-friendly.

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- Pick choice c closest to true value p.

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  - $MSE = (c(\{s\}) - p)^2$?

Agent Goal

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- Question: Does it matter whether objective is
  - $MAE = |c(\{s\}) - p|$, or
  - $MSE = (c(\{s\}) - p)^2$?
- early? eventually (asymptotically)?

(Non-Linear) Algebra Sucks. Optimal C v:

$$\underline{v}^{\star}(EV, C) \equiv \begin{cases} 1/(C+1) & \text{if } EV < 1.5/(C+1) \\ 2/(C+1) & \text{if } 1.5/(C+1) < EV < 2.5/(C+1) \\ \vdots & \vdots \\ C/(C+1) & \text{if } (C - 0.5 \cdot C)/(C+1) < EV \end{cases}$$

$$= \frac{1}{C+1} + \left(\frac{1}{C-1}\right) \cdot \left\{ \sum_{i=1}^{C-1} H\left(EV - \frac{2 \cdot i + 1}{2 \cdot C + 2} \right) \right\}$$

where f_i is binomial pdf and $EV = f(\ldots)$.

Tie-Breaking Rule

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- **FP**: Follow Predecessor
- **FO**: Follow Own
- deterministic vs random.
  - deterministic makes tracking problem a lot easier.
  - with binary signals, agent will be either in an IC or her action will be *fully* invertible under MAE!
- PS: in vstar function, I left tie-breaks open.

Rest is "Easy"

Given S observed signals of which h are H (no aspect of ICs yet), and C choices embedded in $\underline{\overline{v}}^{\star}$

$$\mathrm{MAE}_T(C,S) \equiv \int_{p=0}^{1} p \cdot \left(\sum_{h=0}^{S} f_i(S,i,p) \cdot \left| \underline{\overline{v}}^{\star}(EV(h,S),C) - p \right| \right) dp \ .$$

Again, formula is not for IC, but for observation of h H's in S signals.

Sequential Observable

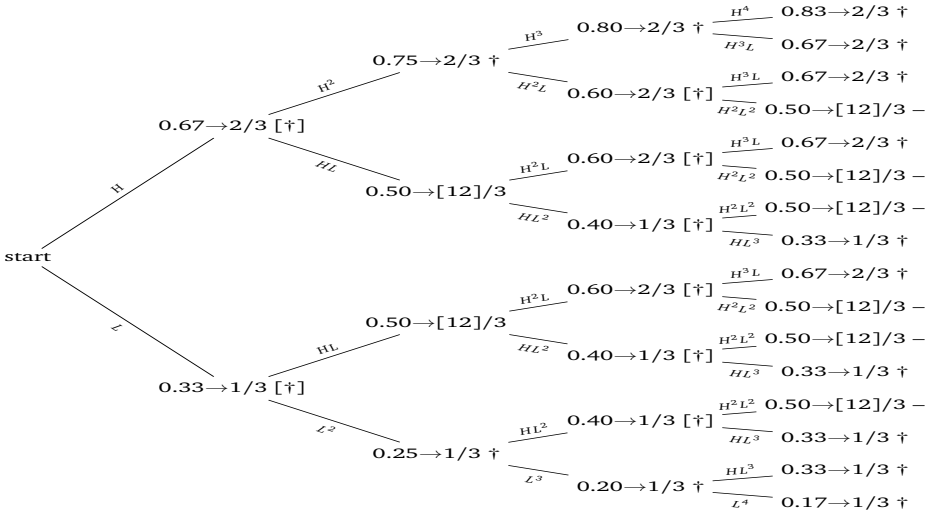
Now we distinguish between

- SigObs
- ActObs (ICs possible)

which means that we can begin to consider ICs.

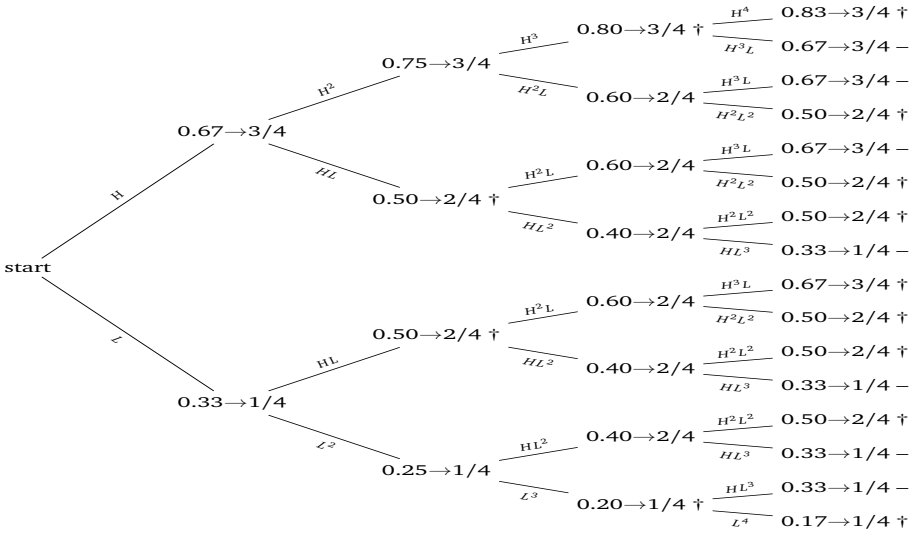
Q: Which TBR is most IC-friendly? unfriendly?

2 Choices: 1/3, 2/3. Indifference: 0.50.



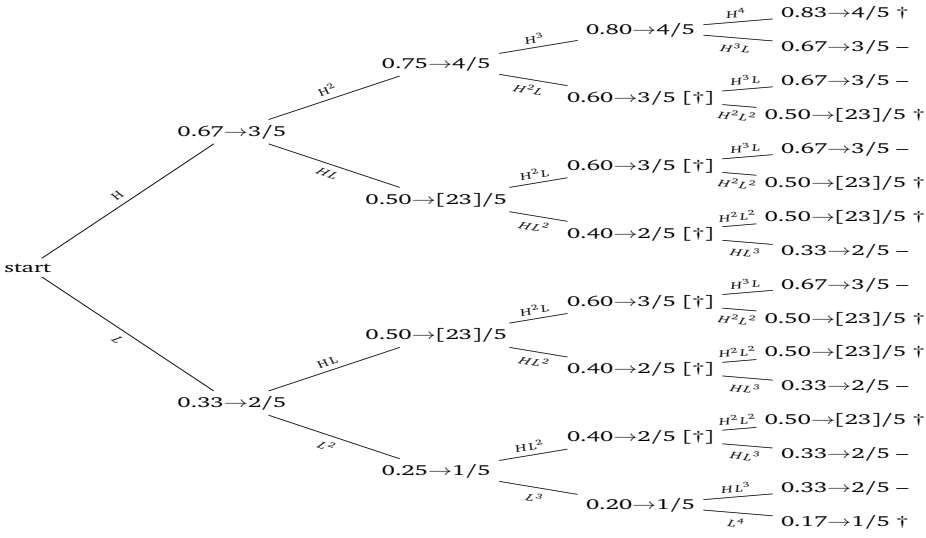
Note: Agent #5 will be responsive iff the 4th agent's inferred value is not 0.50. The choices are spaced narrowly enough that the first agent will have a perfect choice given her signal inference. This is not (necessarily) true for subsequent agents.

3 Choices: 1/4, 1/2, 3/4. Indifference: 0.375, 0.625.



The choices are spaced narrowly enough that the second agent will have a perfect choice given her signal inference.

4 Choices: 1/5, 2/5, 3/5, 4/5. Indifference: 0.3, 0.5, 0.7.



The choices are spaced narrowly enough that the third agent will have a perfect choice given her signal inference.

Maximum (Boring IC) Theorem

- Under FP, IC is guaranteed to start by $2 \cdot C$ agents.
- 4 Choices $\{1/5, 2/5, 3/5, 4/5\}$.
- *HHHLHHH*
  1. 2/3 (action 0.6, not 0.4);
  2. 3/4 (0.8, not 0.6);
  3. 4/5 (0.8, not 0.6);
  4. 4/6 (0.6, not 0.8);
  5. 5/7 (0.8, not 0.6);
  6. 6/8 (0.8, not 0.6);
  7. 7/9 (0.8, not 0.6)
- Agent #8: L: 7/10. H: 8/10. either way, 0.8. IC.

Theoremizor (not Thagomizer)

- compared to action distance now of 1/5
- due to inference distances of 1/10 between L/H

No Max Theorem For FO

- Under FO, the infinitely repeating *HL*|*LH* sequences always return to inference 0.5.
  - cannot guarantee IC onset *ever* (also at 0.375, etc.)
  - nevertheless, ICs happen fast, but not guaranteed within N

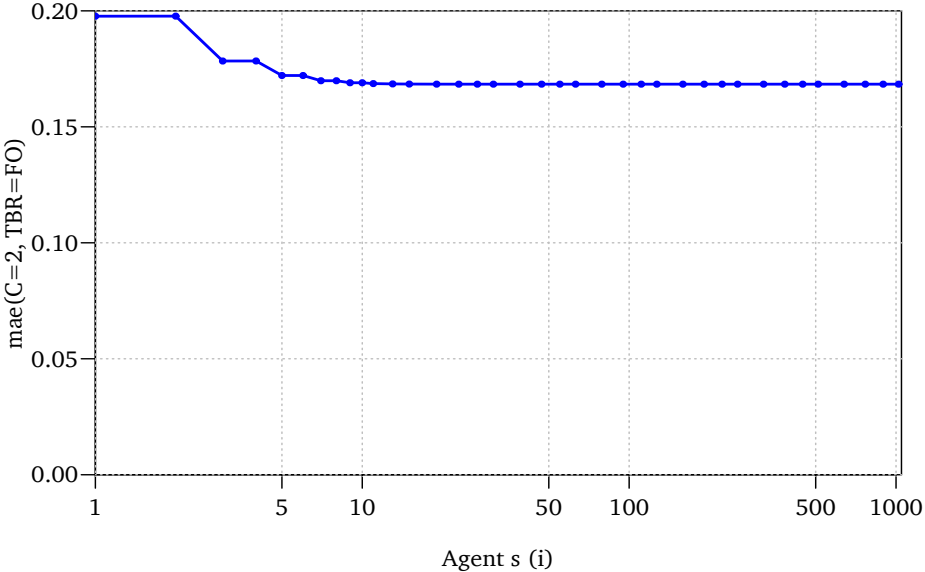
Minimum (Boring IC) Theorem

- Cascade typically do not occur within C agents with best-spaced C (relevant) action choices
  - action $\approx$ signal
  - excellent analogy to think of more choices as signals
  - thus, importance of actions declines with square-root

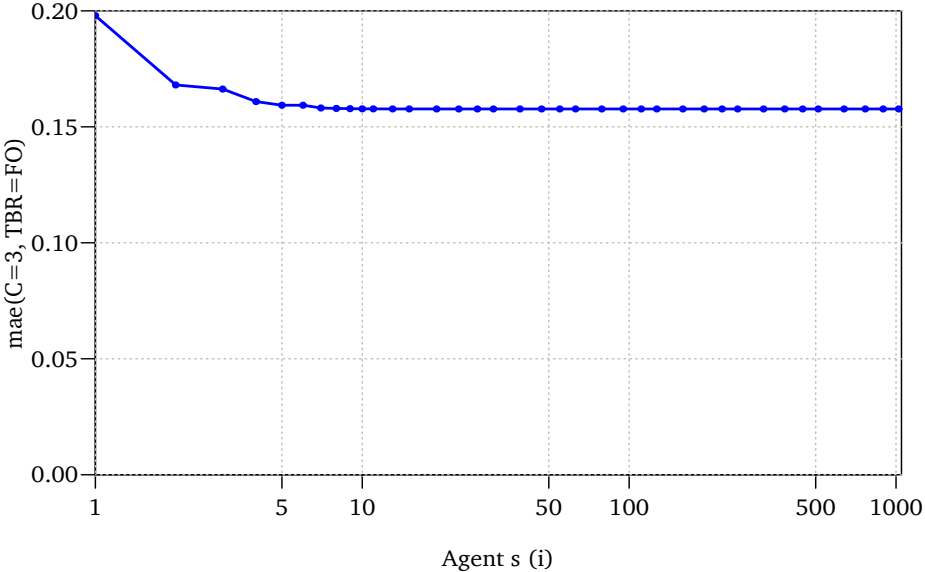
IC / MAE As Function of *C* and *N*

- No easy algebra on non-linear discont. functions.
- Do you care?
  - economic models are for basic insights.
  - IC model is sketch, not realistic. when not (designed to be) realistic, and the goal is exploring basic quantitative aspect, do we really need closed-forms and proofs?
    - Philosophy: is showing basic effect the point?
    - Philosophy: is proving absence of opposite statics important?
  - aesthetic problem, not economic problem
    - yeah, bugs me a little, too!

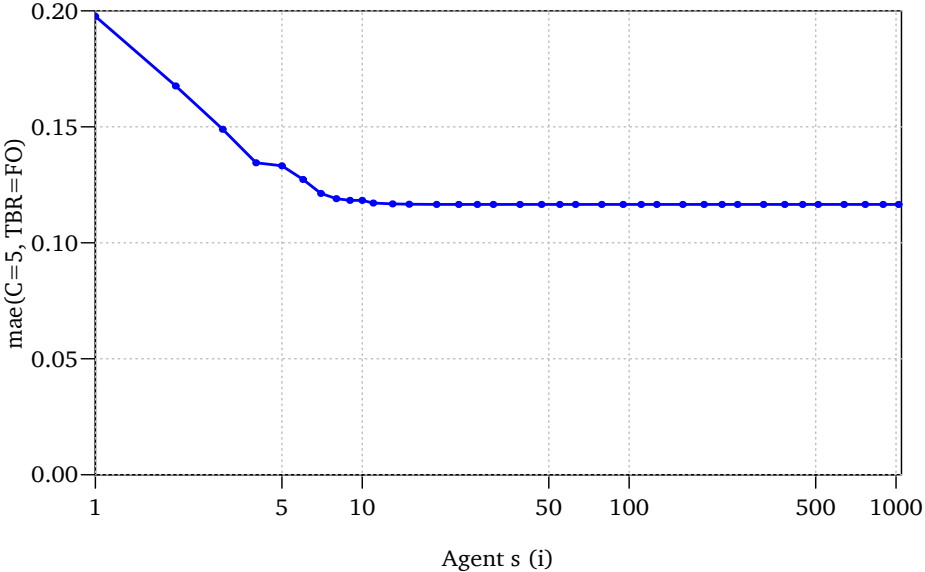
Two Choices (FO) [Expected Mistake]



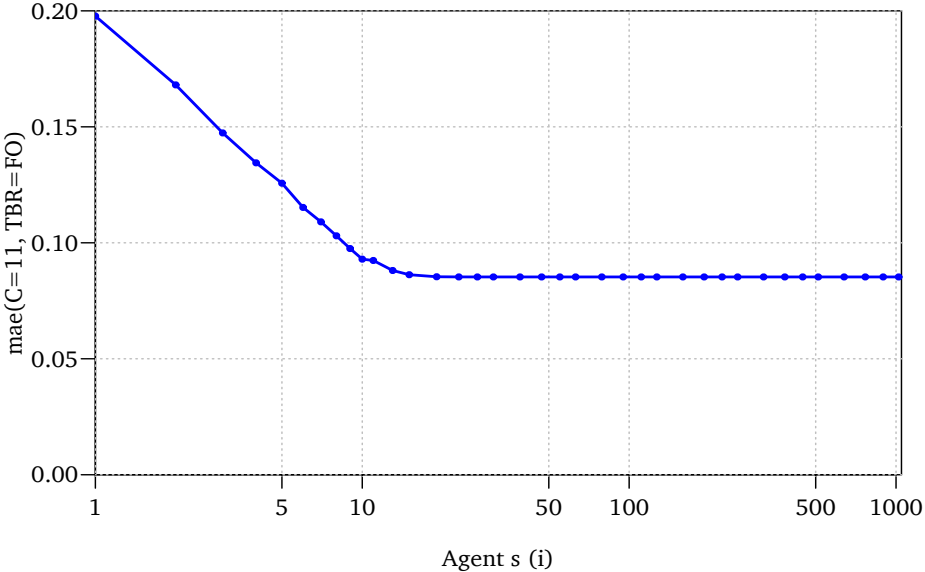
Three Choices (FO)



Five Choices (FO)



Eleven Choices (FO, Thabit)



Conclusion

- ICs are less important, but not unimportant
- Two actions give highest IC relevance.
- A small action space is meaningful but ICs are not critically sensitive.

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Anything Wrong?

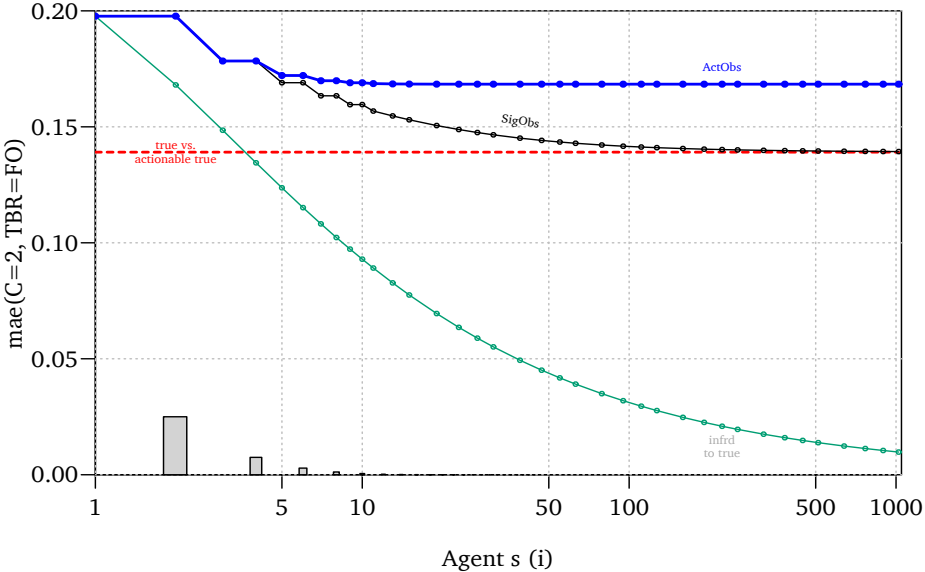
What Benchmark?

- ICs are about information blocking.
  - IC effect: more choices induce later onset due to better invertibility / more responsiveness
- But more choices also make choices closer to truth available.
  - even if all agents had perfect information
  - has really nothing to do with IC invertibility and onset delay effect

“Fair” Benchmark

- What is the IC-specific dampener / reduction?
  - IC invertibility and onset delay effect
- Should we benchmark ActObs against SigObs?!
  - SigObs gains better proximity to truth asymptotically
  - ActObs gains both some more proximity to truth *and* later onset (more invertibility)
  - In relative terms, more action choices could even benefit SigObs *more* than ActObs

Two Choices (FO)



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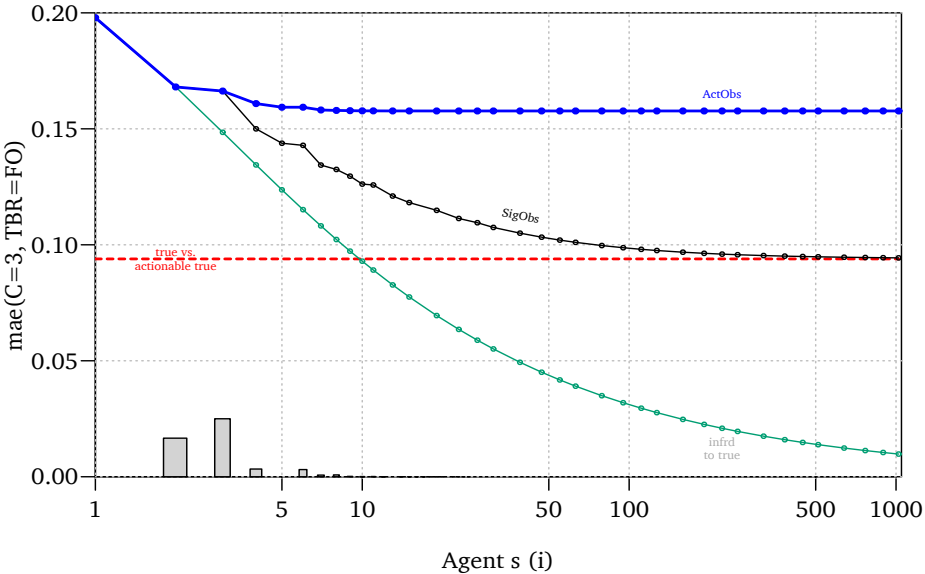

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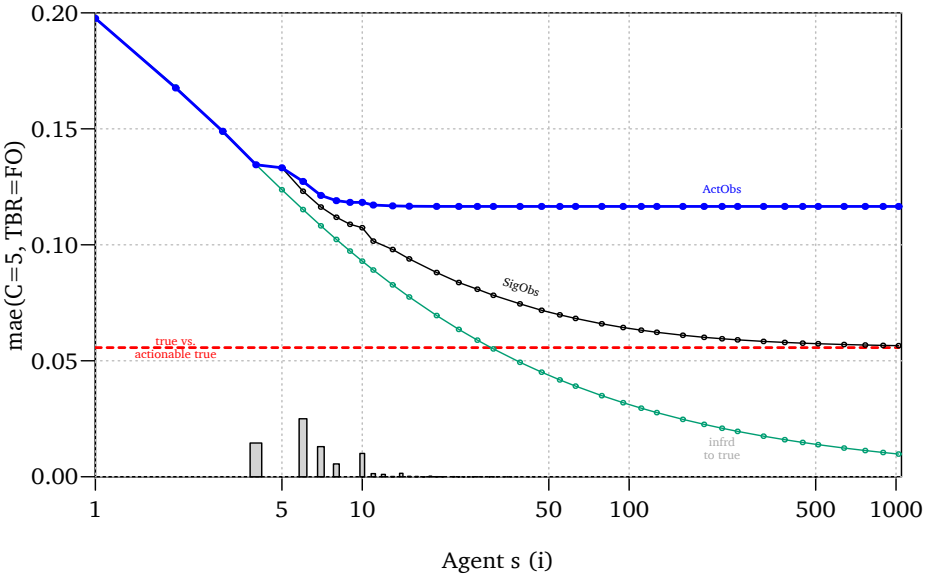

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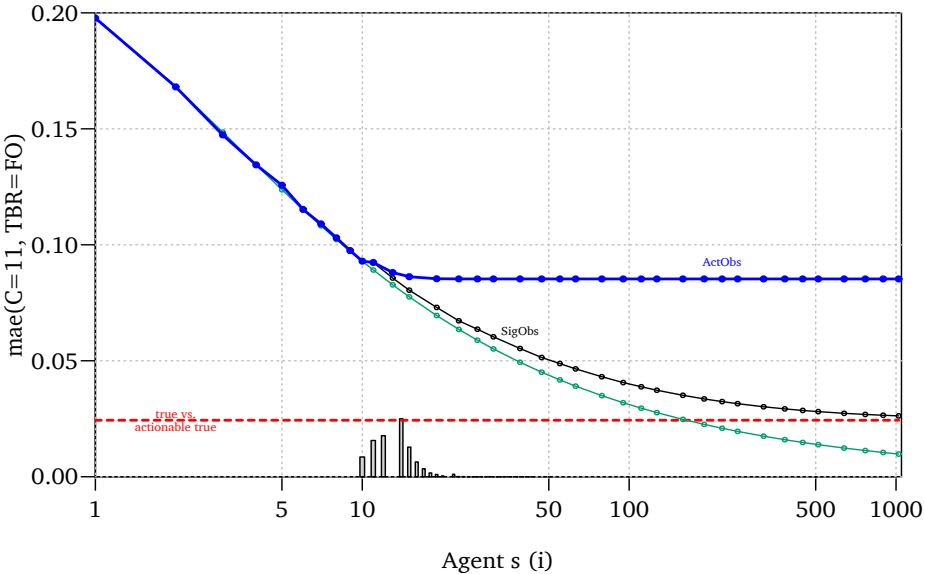
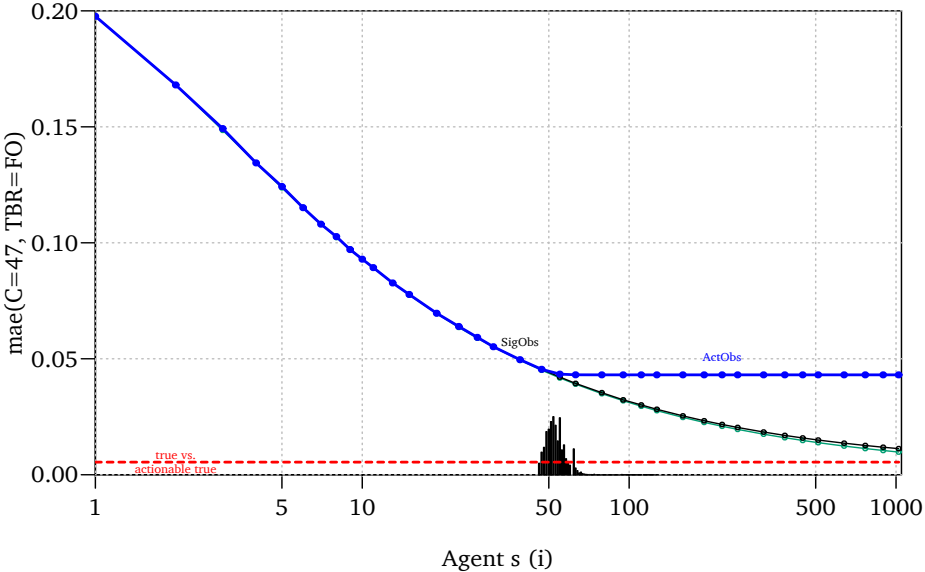
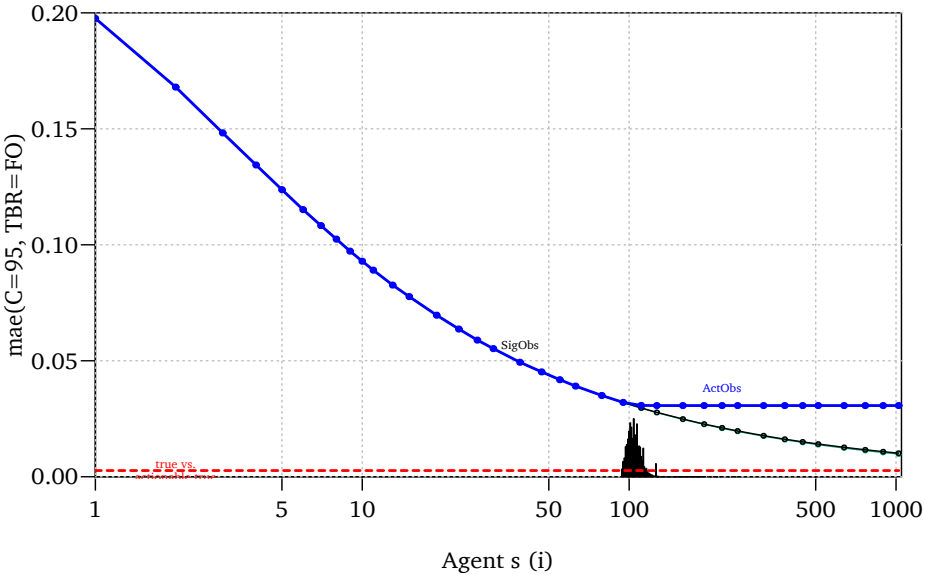


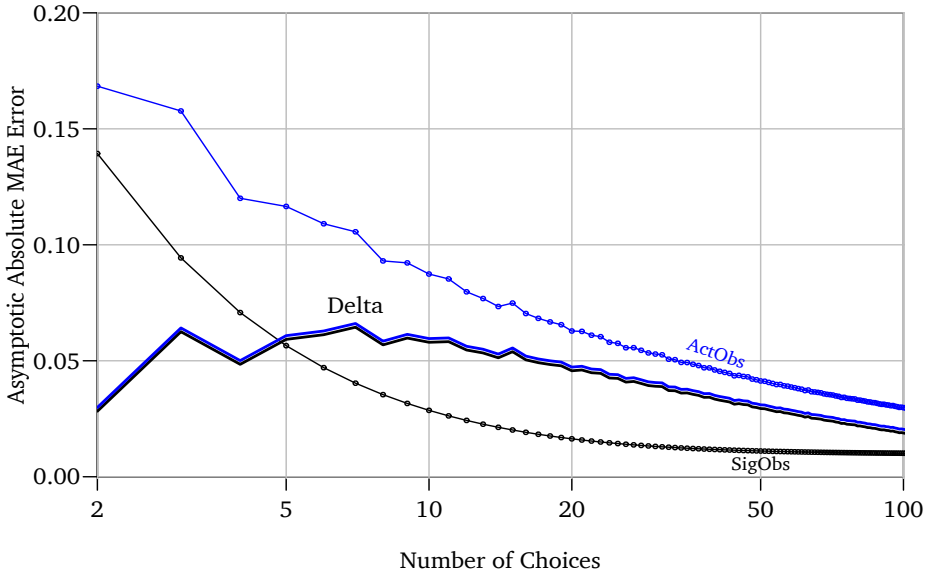
47 Choices (FO)

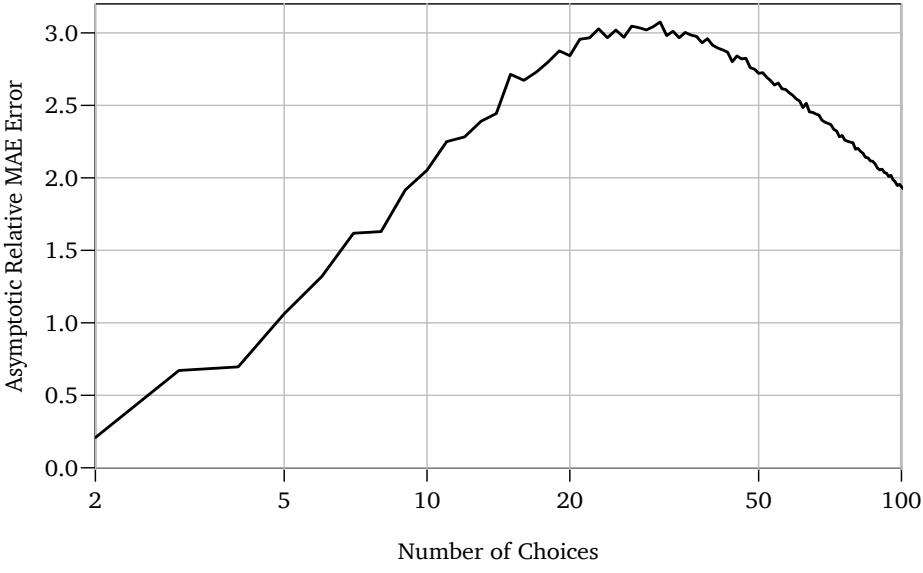


95 Choices (FO)



Relative and Absolute Errors




Asymptotic Relative MAE Error
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
5
10
20
50
100
Number of Choices

Perspective: What Was Interesting?

- What did *I* learn from model?

Perspective: What Was Interesting?

- What did I learn from model?
- More actions matter in the same sense that more information matters:
  - with many signals already, getting more signals becomes ever less important.
  - with many action choices already, getting more action choices becomes ever less important.

- We knew: infinite choices means perfect invertibility
- The importance of ICs is reasonably robust to the number of available actions:
  - In absolute terms (to true best choice), choice availability improvements mix in with invertibility improvements.
  - In relative terms to SigObs, ActObs onset (invertibility) is not even declining after two choices.

Modeling Advice for PhD Students

- Model end result often looks goal-oriented directed, effortless, trivial(?), beautiful(?)
- Often ain't the case.
  - I did not understand needed model ingredients
    - e.g., better uniform than discrete (= 2 weeks). Thabits? stupid?
  - I did not know or understand what paper concluded
    - obvious once explained, not before — that's a good thing!
    - I sometimes ask audiences *before* I tell them the answers to make it clear that it ain't so obvious and effortless, after all.
- Theoremizing is often easier than economizing.
  - Just need first example! Think in numerics first *when possible*.